3.1 Exercises

VOCABULARY CHECK: Fill in the blanks.

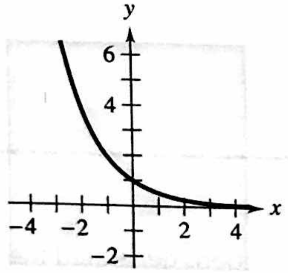
1. Polynomials and rational functions are examples of ________ functions.
2. Exponential and logarithmic functions are examples of nonalgebraic functions, also called ________ functions.
3. The exponential function given by $f(x) = e^x$ is called the ________ ________ function, and the base e is called the ________ base.
4. To find the amount A in an account after t years with principal P and an annual interest rate r compounded n times per year, you can use the formula ________.
5. To find the amount A in an account after t years with principal P and an annual interest rate r compounded continuously, you can use the formula ________.

In Exercises 1–6, evaluate the function at the indicated value of x. Round your result to three decimal places.

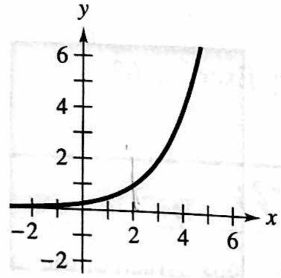
Function	*Value*
1. $f(x) = 3.4^x$	$x = 5.6$
2. $f(x) = 2.3^x$	$x = \frac{3}{2}$
3. $f(x) = 5^x$	$x = -\pi$
4. $f(x) = \left(\frac{2}{3}\right)^{5x}$	$x = \frac{3}{10}$
5. $g(x) = 5000(2^x)$	$x = -1.5$
6. $f(x) = 200(1.2)^{12x}$	$x = 24$

In Exercises 7–10, match the exponential function with its graph. [The graphs are labeled (a), (b), (c), and (d).]

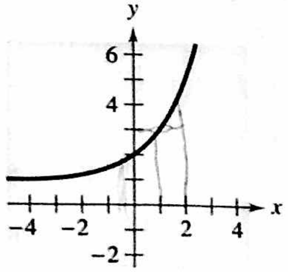
(a)



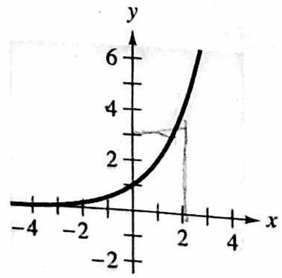
(b)



(c)



(d)



7. $f(x) = 2^x$
8. $f(x) = 2^x + 1$
9. $f(x) = 2^{-x}$
10. $f(x) = 2^{x-2}$

In Exercises 11–16, use a graphing utility to construct a table of values for the function. Then sketch the graph of the function.

11. $f(x) = \left(\frac{1}{2}\right)^x$
12. $f(x) = \left(\frac{1}{2}\right)^{-x}$
13. $f(x) = 6^{-x}$
14. $f(x) = 6^x$
15. $f(x) = 2^{x-1}$
16. $f(x) = 4^{x-3} + 3$

In Exercises 17–22, use the graph of f to describe the transformation that yields the graph of g.

17. $f(x) = 3^x, \quad g(x) = 3^{x-4}$
18. $f(x) = 4^x, \quad g(x) = 4^x + 1$
19. $f(x) = -2^x, \quad g(x) = 5 - 2^x$
20. $f(x) = 10^x, \quad g(x) = 10^{-x+3}$
21. $f(x) = \left(\frac{7}{2}\right)^x, \quad g(x) = -\left(\frac{7}{2}\right)^{-x+6}$
22. $f(x) = 0.3^x, \quad g(x) = -0.3^x + 5$

In Exercises 23–26, use a graphing utility to graph the exponential function.

23. $y = 2^{-x^2}$
24. $y = 3^{-|x|}$
25. $y = 3^{x-2} + 1$
26. $y = 4^{x+1} - 2$

In Exercises 27–32, evaluate the function at the indicated value of x. Round your result to three decimal places.

Function	*Value*
27. $h(x) = e^{-x}$	$x = \frac{3}{4}$
28. $f(x) = e^x$	$x = 3.2$
29. $f(x) = 2e^{-5x}$	$x = 10$
30. $f(x) = 1.5e^{x/2}$	$x = 240$
31. $f(x) = 5000e^{0.06x}$	$x = 6$
32. $f(x) = 250e^{0.05x}$	$x = 20$

In Exercises 33–38, use a graphing utility to construct a table of values for the function. Then sketch the graph of the function.

33. $f(x) = e^x$
34. $f(x) = e^{-x}$
35. $f(x) = 3e^{x+4}$
36. $f(x) = 2e^{-0.5x}$
37. $f(x) = 2e^{x-2} + 4$
38. $f(x) = 2 + e^{x-5}$

In Exercises 39–44, use a graphing utility to graph the exponential function.

39. $y = 1.08^{-5x}$
40. $y = 1.08^{5x}$
41. $s(t) = 2e^{0.12t}$
42. $s(t) = 3e^{-0.2t}$
43. $g(x) = 1 + e^{-x}$
44. $h(x) = e^{x-2}$

In Exercise 45–52, use the One-to-One Property to solve the equation for *x*.

45. $3^{x+1} = 27$
46. $2^{x-3} = 16$
47. $2^{x-2} = \frac{1}{32}$
48. $\left(\frac{1}{5}\right)^{x+1} = 125$
49. $e^{3x+2} = e^3$
50. $e^{2x-1} = e^4$
51. $e^{x^2-3} = e^{2x}$
52. $e^{x^2+6} = e^{5x}$

Compound Interest **In Exercises 53–56, complete the table to determine the balance *A* for *P* dollars invested at rate *r* for *t* years and compounded *n* times per year.**

n	1	2	4	12	365	Continuous
A						

53. $P = \$2500, r = 2.5\%, t = 10$ years
54. $P = \$1000, r = 4\%, t = 10$ years
55. $P = \$2500, r = 3\%, t = 20$ years
56. $P = \$1000, r = 6\%, t = 40$ years

Compound Interest **In Exercises 57–60, complete the table to determine the balance *A* for \$12,000 invested at rate *r* for *t* years, compounded continuously.**

t	10	20	30	40	50
A					

57. $r = 4\%$
58. $r = 6\%$
59. $r = 6.5\%$
60. $r = 3.5\%$

61. ***Trust Fund*** On the day of a child's birth, a deposit of \$25,000 is made in a trust fund that pays 8.75% interest, compounded continuously. Determine the balance in this account on the child's 25th birthday.

62. ***Trust Fund*** A deposit of \$5000 is made in a trust fund that pays 7.5% interest, compounded continuously. It is specified that the balance will be given to the college from which the donor graduated after the money has earned interest for 50 years. How much will the college receive?

63. ***Inflation*** If the annual rate of inflation averages 4% over the next 10 years, the approximate costs C of goods or services during any year in that decade will be modeled by $C(t) = P(1.04)^t$, where t is the time in years and P is the present cost. The price of an oil change for your car is presently \$23.95. Estimate the price 10 years from now.

64. ***Demand*** The demand equation for a product is given by

$$p = 5000\left(1 - \frac{4}{4 + e^{-0.002x}}\right)$$

where p is the price and x is the number of units.

(a) Use a graphing utility to graph the demand function for $x > 0$ and $p > 0$.

(b) Find the price p for a demand of $x = 500$ units.

(c) Use the graph in part (a) to approximate the greatest price that will still yield a demand of at least 600 units.

65. ***Computer Virus*** The number V of computers infected by a computer virus increases according to the model $V(t) = 100e^{4.6052t}$, where t is the time in hours. Find (a) $V(1)$, (b) $V(1.5)$, and (c) $V(2)$.

66. ***Population*** The population P (in millions) of Russia from 1996 to 2004 can be approximated by the model $P = 152.26e^{-0.0039t}$, where t represents the year, with $t = 6$ corresponding to 1996. (Source: Census Bureau, International Data Base)

(a) According to the model, is the population of Russia increasing or decreasing? Explain.

(b) Find the population of Russia in 1998 and 2000.

(c) Use the model to predict the population of Russia in 2010.

67. ***Radioactive Decay*** Let Q represent a mass of radioactive radium (^{226}Ra) (in grams), whose half-life is 1599 years. The quantity of radium present after t years is $Q = 25\left(\frac{1}{2}\right)^{t/1599}$.

(a) Determine the initial quantity (when $t = 0$).

(b) Determine the quantity present after 1000 years.

(c) Use a graphing utility to graph the function over the interval $t = 0$ to $t = 5000$.

68. ***Radioactive Decay*** Let Q represent a mass of carbon 14 (^{14}C) (in grams), whose half-life is 5715 years. The quantity of carbon 14 present after t years is $Q = 10\left(\frac{1}{2}\right)^{t/5715}$.

(a) Determine the initial quantity (when $t = 0$).

(b) Determine the quantity present after 2000 years.

(c) Sketch the graph of this function over the interval $t = 0$ to $t = 10{,}000$.